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Computing multiplicative topological indices of some chemical nenotubes and networks

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Abstract: The aim of this paper is to calculate the multiplicative topological indices of Zigzag polyhex nanotubes, Armchair polyhex nanotubes, Carbon nanocone networks, two dimensional Silicate network, Chain silicate network, six dimensional Hexagonal network, five dimensional Oxide network and four dimensional Honeycomb network.

Keywords: Chemical graph theory, multiplicative topological index, degree.

MSC: 05C05, 05C12, 05C50.

1. Introduction

Nowadays graph theory is one of the most ironic and cited branch of mathematics due to its direct applications in our daily life. It is widely used in Computer networking and Chemistry. The area of graph theory related to Chemistry known as Chemical graph theory. This term firstly introduced by Balaban in book [1] in 1976. After it in 1991, Bonchev discussed more concepts in book [2] and in the book [3] by Trinajstić, we found a facet ideas about chemical graph theory its uses and applications in our daily life.

In the recent few years, lot of work has been done in chemical graph theory like in [4], Ali *et al.* calculated the topological indices of some chemical compounds. Pattabiraman and Suganya in [5] and Kanabur in [6] calculated topological indices of some well known graphs. The concept of multiplicative topological indices of graphs was given in [7–10]. In [11], Kahasy *et al.* calculate atom bond connectivity temperature index of some important organic compounds. Topological indices of some families of nanostar have been calculated in [12]. He and Jiang, in [13] calculated degree resistance distance of some trees. Degree-based multiplicative Atom-bond Connectivity index of some Nanostructures has been discussed [14]. In 2018, Hussain and Sabar [15] calculated multiplicative topological indices of single-walled titania nanotube. In [16], Kulli calculated some topological indices of two dimensional Silicate network, Chain silicate network, six dimensional Hexagonal network, five dimensional Oxide network and four dimensional Honeycomb network. Recently, Kulli [17] computed some topological indices of Zigzag polyhex nanotubes, Armchair polyhex nanotubes and Carbon nanocone networks. The main motivation of this work directly came from the papers [15–17].

2. Preliminaries

Let $G(V(G), E(G))$ be a finite, simple and connected graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$ is the set of vertices and $E(G) = \{e_1, e_2, \ldots, e_m\}$ is the set of edges among the vertices of the graph. Consider a geodesic metric $d_G : V(G) \times V(G) \to \mathbb{R}$ defined as $d_G(u, v)$ is the number of edges between u and v in shortest path for any $u, v \in V(G)$. All the vertices which are exactly at distance 1 from $u \in V(G)$ are neighborhoods of u in graph G and collection of that vertices is called neighborhood set of u in graph G written as $N_G(u)$. Cardinality of neighborhood set of $u \in V(G)$ is called degree of u in graph G and in this paper it is denoted as ξ_u.

First multiplicative Zagreb index is defined as:

$$II_1^*(G) = \prod_{rt \in E(G)} (\xi_r + \xi_t). \tag{1}$$

Second multiplicative Zagreb index is defined as:

$$II_2(G) = \prod_{rt \in E(G)} (\xi_r \,.\, \xi_t). \tag{2}$$

The multiplicative first and second hyper-Zagreb indices are defined as:

$$HII_1(G) = \prod_{rt \in E(G)} (\xi_r + \xi_t)^2. \tag{3}$$

$$HII_2(G) = \prod_{rt \in E(G)} (\xi_r \,.\, \xi_t)^2. \tag{4}$$

First and second multiplicative generalized Zagreb indices are the generalized form of first and second multiplicative Zagreb indices as well as first and second multiplicative hyper-Zagreb indices. First and second multiplicative generalized Zagreb indices are defined as:

$$MZ_1^{\alpha}(G) = \prod_{rt \in E(G)} (\xi_r + \xi_t)^{\alpha}. \tag{5}$$

$$MZ_2^{\alpha}(G) = \prod_{rt \in E(G)} (\xi_r \,.\, \xi_t)^{\alpha}. \tag{6}$$

Multiplicative sum and product connectivity indices are defined as:

$$SCII(G) = \prod_{rt \in E(G)} \frac{1}{\sqrt{\xi_r + \xi_t}}. \tag{7}$$

$$PCII(G) = \prod_{rt \in E(G)} \frac{1}{\sqrt{\xi_r \,.\, \xi_t}}. \tag{8}$$

The multiplicative atomic bond connectivity index and geometric arithmetic index are defined as:

$$ABCII(G) = \prod_{rt \in E(G)} \sqrt{\frac{\xi_r + \xi_t - 2}{\xi_r \,.\, \xi_t}}. \tag{9}$$

$$G^*AII(G) = \prod_{rt \in E(G)} \left(\frac{2\sqrt{\xi_r \,.\, \xi_t}}{\xi_r + \xi_t} \right). \tag{10}$$

The general multiplicative geometric arithmetic index is defined as:

$$G^*A^{\alpha}II(G) = \prod_{rt \in E(G)} \left(\frac{2\sqrt{\xi_r \,.\, \xi_t}}{\xi_r + \xi_t} \right)^{\alpha}. \tag{11}$$

Fact 1. *Let $\eta_1, \eta_2, \ldots, \eta_n$ be a sequence. Then*

$$\prod_{i=1}^{n} (\eta_i)^{\gamma} = \left(\prod_{i=1}^{n} \eta_i \right)^{\gamma},$$

where γ is a constant.

Proposition 2. *Let $G(V(G), E(G))$ be any simple, connected and finite graph. Then*

$$SCII(G) = (II_1^*(G))^{-\frac{1}{2}}.$$

Proof.

$$\begin{aligned} SCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\xi_r + \xi_t}} \\ &= \left(\frac{1}{\sqrt{\xi_r + \xi_t}}\right)^{|E(G)|} \\ &= (\xi_r + \xi_t)^{-\frac{|E(G)|}{2}} \\ &= \left(\prod_{rt\in E(G)} (\xi_r + \xi_t)\right)^{-\frac{1}{2}} \\ &= (II_1^*(G))^{-\frac{1}{2}}. \end{aligned}$$

□

Proposition 3. *Let $G(V(G), E(G))$ be any simple, connected and finite graph. Then*

$$PCII(G) = (II_2(G))^{-\frac{1}{2}}.$$

Proof.

$$\begin{aligned} PCII(G) &= \prod_{rt\in E(G)} \frac{1}{\sqrt{\xi_r \cdot \xi_t}} \\ &= \left(\frac{1}{\sqrt{\xi_r \cdot \xi_t}}\right)^{|E(G)|} \\ &= (\xi_r \cdot \xi_t)^{-\frac{|E(G)|}{2}} \\ &= \left(\prod_{rt\in E(G)} (\xi_r \cdot \xi_t)\right)^{-\frac{1}{2}} \\ &= (II_2(G))^{-\frac{1}{2}}. \end{aligned}$$

□

For detail concepts of topological indices on graphs we refer [10,15].

3. Main results

3.1. Zigzag Polyhex Nanotubes

Zigzag polyhex nantube is denoted as $TUZC_6[p,q]$, where p is the number of hexagons in a row and q is the number of hexagons in a column. A $2-$Dimensional networks of $TUZC_6[p,q]$ is shown in Figure 1. Let $G(V(G), E(G))$ be the graph of a (p,q) dimensional zigzag polyhex nantube. It is easy to check that $|V(G)| = 2p(q+1)$ and $|E(G)| = p(3q+2)$. In this structure there are two types of edges on the basis of their degrees, so we can decompose the the set of edges as $E(G) = E_1(G) \bigcup E_2(G)$, where

$$\begin{aligned} E_1(G) &= \{e = rt \in E(G) \mid \xi_r = 2, \xi_t = 3\}, \\ E_2(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 3\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = 4p$ and $|E_2(G)| = p(3q-2)$.

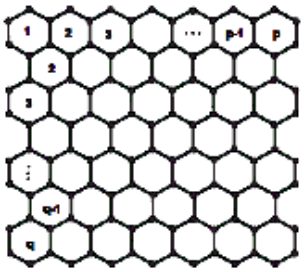

Figure 1. 2−Dimensional networks of $TUZC_6[p,q]$

Theorem 4. *Let $G(V(G), E(G))$ be the graph of Zigzag polyhex nanotube $TUZC_6[p,q]$. Then the first multiplicative Zagreb index for G is $2^{p(3q-2)} \times 3^{p(3q-2)} \times 5^{4p}$.*

Proof. The first multiplicative Zagreb index is:

$$\begin{aligned} II_1^*(G) &= \prod_{rt\in E(G)} (\xi_r + \xi_t) \\ &= \prod_{rt\in E_1(G)} (\xi_r + \xi_t) \times \prod_{rt\in E_2(G)} (\xi_r + \xi_t) \\ &= (2+3)^{|E_1(G)|} \times (3+3)^{|E_2(G)|} \\ &= 5^{4p} \times 2^{3pq-2p} \times 3^{3pq-2p} \\ &= 2^{p(3q-2)} \times 3^{p(3q-2)} \times 5^{4p}. \end{aligned}$$

□

Theorem 5. *Let $G(V(G), E(G))$ be the graph of Zigzag polyhex nanotube $TUZC_6[p,q]$. Then the second multiplicative Zagreb index for G is $2^{4p} \times 3^{6pq}$.*

Proof. The second multiplicative Zagreb index is:

$$\begin{aligned} II_2(G) &= \prod_{rt\in E(G)} (\xi_r \cdot \xi_t) \\ &= \prod_{rt\in E_1(G)} (\xi_r \cdot \xi_t) \times \prod_{rt\in E_2(G)} (\xi_r \cdot \xi_t) \\ &= (2 \cdot 3)^{|E_1(G)|} \times (3 \cdot 3)^{|E_2(G)|} \\ &= 2^{4p} \times 3^{4p} \times 3^{6pq-4p} \\ &= 2^{4p} \times 3^{6pq}. \end{aligned}$$

□

Theorem 6. *Let $G(V(G), E(G))$ be the graph of Zigzag polyhex nanotube $TUZC_6[p,q]$. Then the multiplicative atomic bond connectivity index for G is $2^{p(3q-4)} \times 3^{p(2-3q)}$.*

Proof. The multiplicative atomic bond connectivity index is:

$$\begin{aligned} ABCII(G) &= \prod_{rt\in E(G)} \sqrt{\frac{\xi_r + \xi_t - 2}{\xi_r \cdot \xi_t}} \\ &= \prod_{rt\in E_1(G)} \sqrt{\frac{\xi_r + \xi_t - 2}{\xi_r \cdot \xi_t}} \times \prod_{rt\in E_2(G)} \sqrt{\frac{\xi_r + \xi_t - 2}{\xi_r \cdot \xi_t}} \end{aligned}$$

$$\begin{aligned} &= \left(\frac{2+3-2}{2\,.\,3}\right)^{\frac{|E_1(G)|}{2}} \times \left(\frac{3+3-2}{3\,.\,3}\right)^{\frac{|E_2(G)|}{2}} \\ &= \left(\frac{1}{2}\right)^{2p} \times \left(\frac{2}{3}\right)^{p(3q-2)} \\ &= 2^{p(3q-4)} \times 3^{p(2-3q)}. \end{aligned}$$

□

Theorem 7. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Zigzag polyhex nanotube $TUZC_6\left[p,q\right]$. Then the multiplicative geometric arithmetic index for G is $2^{6p}\times 3^{2p}\times 5^{-4p}$.*

Proof. The multiplicative geometric arithmetic index is:

$$\begin{aligned} G^{*}AII(G) &= \prod_{rt\in E(G)}\left(\frac{2\sqrt{\xi_r\cdot\xi_t}}{\xi_r+\xi_t}\right) \\ &= \prod_{rt\in E_1(G)}\left(\frac{2\sqrt{\xi_r\cdot\xi_t}}{\xi_r+\xi_t}\right)\times\prod_{rt\in E_2(G)}\left(\frac{2\sqrt{\xi_r\cdot\xi_t}}{\xi_r+\xi_t}\right) \\ &= \left(\frac{2\sqrt{2\,.\,3}}{2+3}\right)^{|E_1(G)|}\times\left(\frac{2\sqrt{3\,.\,3}}{3+3}\right)^{|E_2(G)|} \\ &= \left(\frac{2\,.\,2^{\frac{1}{2}}\,.\,3^{\frac{1}{2}}}{5}\right)^{4p}\times\left(\frac{2\,.\,3}{3+3}\right)^{2pq-2p} \\ &= 2^{6p}\times 3^{2p}\times 5^{-4p}. \end{aligned}$$

□

3.2. Armchair Polyhex Nanotubes

Carbon polyhex nantubes are those nantubes in which the cylindrical surface is entirely made up of hexagons. These type of carbon nantubes have very interesting thermal, electrical and mechanical properties, actually these are very stabile in nature. A $2-$dimensional networks of $TUAC_6\left[p,q\right]$ is shown in Figure 2. Let

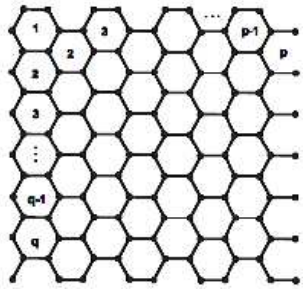

Figure 2. $2-$dimensional networks of $TUAC_6\left[p,q\right]$

$G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Armchair polyhex nanotubes. It is easy to check that $|V\left(G\right)|=2p\left(q+1\right)$ and $|E\left(G\right)|=p\left(3q+2\right)$. In this structure there are three types of edges on the basis of their degrees, so we can decompose the the set of edges as $E\left(G\right)=E_1\left(G\right)\bigcup E_2\left(G\right)\bigcup E_3\left(G\right)$, where

$$\begin{aligned} E_1\left(G\right) &= \left\{e=rt\in E\left(G\right)\ \mid\ \xi_r=2,\xi_t=2\right\}, \\ E_2\left(G\right) &= \left\{e=rt\in E\left(G\right)\ \mid\ \xi_r=2,\xi_t=3\right\}, \\ E_3\left(G\right) &= \left\{e=rt\in E\left(G\right)\ \mid\ \xi_r=3,\xi_t=3\right\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = p$, $|E_2(G)| = 2p$ and $|E_3(G)| = p(3q-1)$. From this edge parttion, we can easily obtain the following results.

Theorem 8. *Let $G(V(G), E(G))$ be the graph of Armchair polyhex nanotube $TUAC_6[p,q]$. Then the first multiplicative Zagreb index for G is $2^{p(1+3q)} \times 3^{p(3q-1)} \times 5^{2p}$.*

Theorem 9. *Let $G(V(G), E(G))$ be the graph of Armchair polyhex nanotube $TUAC_6[p,q]$. Then the second multiplicative Zagreb index for G is $2^{4p} \times 3^{6pq}$.*

Theorem 10. *Let $G(V(G), E(G))$ be the graph of Armchair polyhex nanotube $TUAC_6[p,q]$. Then the multiplicative atomic bond connectivity index for G is $\sqrt{2^{p(6q-5)}} \times 3^{p(1-3q)}$.*

Theorem 11. *Let $G(V(G), E(G))$ be the graph of Armchair polyhex nanotube $TUAC_6[p,q]$. Then the multiplicative geometric arithmetic index for G is $2^{3p} \times 3^p \times 5^{-2p}$.*

3.3. Carbon Nanocone Networks

An $n-$dimensional one$-$pentagone nanocone is denoted as $CNC_5[n]$, where n is the number of hexagons layers encompassing the conical surface of nanocone and 5 denotes that there is a pentagon on the tip called its core. A $6-$dimensional one$-$pentagonal nanocone network is shown in the Figure 3. Now, $|V(G)| = 5(n+1)^2$ and $|E(G)| = 5\left(\frac{3}{2}n^2 + \frac{5}{2}n + 1\right)$. In this structure there are following three types of edges: $E(G) = E_1(G) \bigcup E_2(G) \bigcup E_3(G)$, where

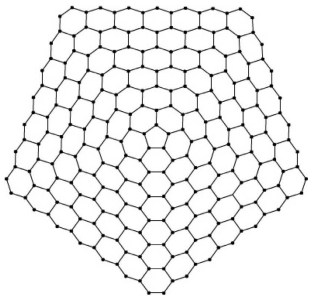

Figure 3. $6-$dimensional one$-$pentagonal nanocone network

$$\begin{aligned} E_1(G) &= \{e = rt \in E(G) \mid \xi_r = 2, \xi_t = 2\}, \\ E_2(G) &= \{e = rt \in E(G) \mid \xi_r = 2, \xi_t = 3\}, \\ E_3(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 3\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = 5$, $|E_2(G)| = 10n$ and $|E_3(G)| = 5\left(\frac{3}{2}n^2 + \frac{1}{2}n\right)$ and following results can be obtained immediately.

Theorem 12. *Let $G(V(G), E(G))$ be the graph of Carbon nanocone networks $CNC_5[n]$. Then the first multiplicative Zagreb index for G is $\sqrt{2^{15n^2+5n+20}} \times \sqrt{3^{15n^2+5n}} \times 5^{10n}$.*

Theorem 13. *Let $G(V(G), E(G))$ be the graph of be the graph of Carbon nanocone networks $CNC_5[n]$. Then the second multiplicative Zagreb index for G is $2^{10(n+1)} \times 3^{15n(n+1)}$.*

Theorem 14. *Let $G(V(G), E(G))$ be the graph of Carbon nanocone networks $CNC_5[n]$. Then the multiplicative atomic bond connectivity index for G is $\sqrt{2^{5(3n^2-n-1)}} \times \sqrt{3^{-5n(3n+1)}}$.*

Theorem 15. *Let $G(V(G), E(G))$ be the graph of Carbon nanocone networks $CNC_5[n]$. Then the multiplicative geometric arithmetic index for G is $2^{15n} \times 3^{5n} \times 5^{-10n}$.*

3.4. Silicate Networks

Silicates are formed by mixing of metal carbonates or metal oxides with sand. Silicate network is denoted as SL_n, where n is the number of hexagons between the center and boundary of SL_n. A $2-$dimensional

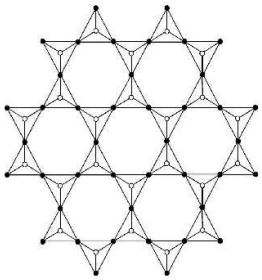

Figure 4. $2-$dimensional silicate network

silicate network is shown in Figure 4. Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Silicate networks, then $\left|V\left(G\right)\right| = 3n\left(5n+1\right)$ and $\left|E\left(G\right)\right| = 36n^2$. In this structure there are three types of edges on the basis of their degrees, so we can decompose the the set of edges as $E\left(G\right) = E_1\left(G\right)\bigcup E_2\left(G\right)\bigcup E_3\left(G\right)$, where

$$\begin{aligned} E_1\left(G\right) &= \left\{e = rt \in E\left(G\right) \mid \xi_r = 3, \xi_t = 3\right\}, \\ E_2\left(G\right) &= \left\{e = rt \in E\left(G\right) \mid \xi_r = 3, \xi_t = 6\right\}, \\ E_3\left(G\right) &= \left\{e = rt \in E\left(G\right) \mid \xi_r = 6, \xi_t = 6\right\}. \end{aligned}$$

Now, $\left|E_1\left(G\right)\right| = 6n$, $\left|E_2\left(G\right)\right| = 6n\left(3n+1\right)$ and $\left|E_3\left(G\right)\right| = 6n\left(3n-2\right)$, so we have following results.

Theorem 16. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Silicate network SL_n. Then the first multiplicative Zagreb index for G is $2^{18n(2n-1)} \times 3^{6n(9n+1)}$.*

Theorem 17. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Silicate network SL_n. Then the second multiplicative Zagreb index for G is $2^{18n(3n-1)} \times 3^{72n^2}$.*

Theorem 18. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Silicate network SL_n. Then the multiplicative atomic bond connectivity index for G is $2^{-9n(2n-1)} \times 3^{-36n^2} \times 5^{3n(3n-2)} \times 7^{3n(3n+1)}$.*

Theorem 19. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Silicate network SL_n. Then the multiplicative geometric arithmetic index for G is $2^{9n(3n+1)} \times 3^{-6n(3n+1)}$.*

3.5. Chain Silicate Networks

Chain is obtained by arranging n tetrahedral linearly. Chain silicate networks are denoted as CS_n.

Figure 5. Chain silicate network

A chain silicate network is shown in Figure 5. Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of chain Silicate networks,

then $|V(G)| = 3n+1$ and $|E(G)| = 6n$. In this structure there are three types of edges on the basis of their degrees, so we can decompose the the set of edges as $E(G) = E_1(G) \bigcup E_2(G) \bigcup E_3(G)$, where

$$\begin{aligned} E_1(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 3\}, \\ E_2(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 6\}, \\ E_3(G) &= \{e = rt \in E(G) \mid \xi_r = 6, \xi_t = 6\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = n+4$, $|E_2(G)| = 2(2n-1)$ and $|E_3(G)| = n-2$.

Theorem 20. *Let $G(V(G), E(G))$ be the graph of Chain Silicate networks CS_n. Then the first multiplicative Zagreb index for G is $2^{3n} \times 3^{2(5n-1)}$.*

Theorem 21. *Let $G(V(G), E(G))$ be the graph of Chain Silicate networks CS_n. Then the second multiplicative Zagreb index for G is $2^{6(n-1)} \times 3^{12n}$.*

Theorem 22. *Let $G(V(G), E(G))$ be the graph of Chain Silicate networks CS_n. Then the multiplicative atomic bond connectivity index for G is $\sqrt{2^{-3n+12}} \times \sqrt{3^{-12n}} \times \sqrt{5^{n-2}} \times 7^{2n-1}$.*

Theorem 23. *Let $G(V(G), E(G))$ be the graph of Chain Silicate networks CS_n. Then the multiplicative geometric arithmetic index for G is $2^{3(2n-1)} \times 3^{-2(2n-1)}$.*

3.6. Hexagonal Networks

It is known that there exist three regular plane tailings with composition of same kind of regular polygons such as triangles, squares and hexagonal. Triangular tiling is used in the construction of hexagonal networks. hexagonal network is denoted as HX_n, where n is the number of vertices of in each side of hexagon.

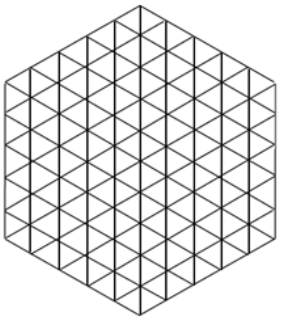

Figure 6. 6−diminsional hexagonal network

A 6−dimensional hexagonal network is shown in Figure 6. Let $G(V(G), E(G))$ be the graph of hexagonal network, then $|V(G)| = 3n^2 - 3n + 1$ and $|E(G)| = 3(3n^2 - 5n + 2)$. In this structure there are five type of edges on the basis of their degrees, so we can decompose the the set of edges as $E(G) = E_1(G) \bigcup E_2(G) \bigcup E_3(G) \bigcup E_4(G) \bigcup E_5(G)$, where

$$\begin{aligned} E_1(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 4\}, \\ E_2(G) &= \{e = rt \in E(G) \mid \xi_r = 3, \xi_t = 6\}, \\ E_3(G) &= \{e = rt \in E(G) \mid \xi_r = 4, \xi_t = 4\}, \\ E_4(G) &= \{e = rt \in E(G) \mid \xi_r = 4, \xi_t = 6\}, \\ E_5(G) &= \{e = rt \in E(G) \mid \xi_r = 6, \xi_t = 6\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = 12$, $|E_2(G)| = 6$, $|E_3(G)| = 6(n-3)$, $|E_4(G)| = 12(n-2)$ and $|E_5(G)| = 3(3n^2 - 11n + 10)$.

Theorem 24. *Let $G(V(G), E(G))$ be the graph of Hexagonal Network HX_n. Then the first multiplicative Zagreb index for G is $2^{18\left(n^2-2n-1\right)} \times 3^{3\left(3n^2-11n+14\right)} \times 5^{12(n-2)} \times 7^{12}$.*

Theorem 25. *Let $G(V(G), E(G))$ be the graph of Hexagonal Network HX_n. Then the second multiplicative Zagreb index for G is $2^{6\left(3n^2-n-9\right)} \times 3^{6\left(3n^2-9n+10\right)}$.*

Theorem 26. *Let $G(V(G), E(G))$ be the graph of Hexagonal Network HX_n. Then the multiplicative atomic bond connectivity index for G is $\sqrt{2^{-3(3n^2-5n+2)}} \times 3^{-3\left(3n^2-10n+13\right)} \times \sqrt{5^{3(3n^2-11n+14)}} \times 7^3$.*

Theorem 27. *Let $G(V(G), E(G))$ be the graph of Hexagonal Network HX_n. Then the multiplicative geometric arithmetic index for G is $2^{3(6n-1)} \times 3^{6(n-2)} \times 5^{-12(n-2)} \times 7^{-12}$.*

3.7. Oxide Networks

An oxide network is denoted as OX_n, where n is the number of dimensions. A $5-$diminsional oxide

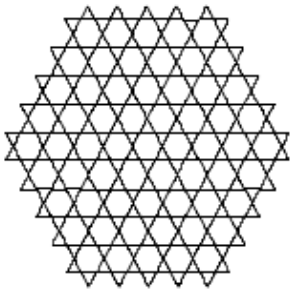

Figure 7. $5-$diminsional oxide network

network is shown in Figure 7. Let $G(V(G), E(G))$ be the graph of oxide network, then $|V(G)| = 9n^2 + 3n$ and $|E(G)| = 18n^2$. In this structure there are two type of edges on the basis of their degrees, so we can decompose the the set of edges such as $E(G) = E_1(G) \bigcup E_2(G)$, where

$$\begin{aligned} E_1(G) &= \{e = rt \in E(G) \mid \xi_r = 2, \xi_t = 4\}, \\ E_2(G) &= \{e = rt \in E(G) \mid \xi_r = 4, \xi_t = 4\}. \end{aligned}$$

It is easy to check that $|E_1(G)| = 12n$ and $|E_2(G)| = 6n(3n-2)$.

Theorem 28. *Let $G(V(G), E(G))$ be the graph of Oxide Network OX_n. Then the first multiplicative Zagreb index for G is $2^{6n(9n-4)} \times 3^{12n}$.*

Theorem 29. *Let $G(V(G), E(G))$ be the graph of Oxide Network OX_n. Then the second multiplicative Zagreb index for G is $2^{12n(6n-1)}$.*

Theorem 30. *Let $G(V(G), E(G))$ be the graph of Oxide Network OX_n. Then the multiplicative atomic bond connectivity index for G is $2^{-3n(9n-4)} \times 3^{3n(3n-2)}$.*

Theorem 31. *Let $G(V(G), E(G))$ be the graph of Oxide Network OX_n. Then the multiplicative geometric arithmetic index for G is $2^{18n} \times 3^{-12n}$.*

3.8. Honeycomb Networks

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. A honeycomb is denoted as HC_n, where n is the number of hexagons between the central and boundary hexagon.

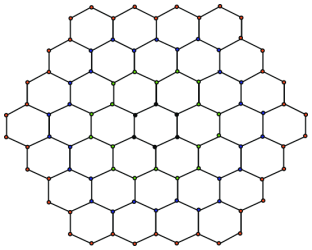

Figure 8. 4−diminsional honeycomb network

A 4−dimensional honeycomb network is shown in Figure 8. Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of honeycomb network, then $\left|V\left(G\right)\right|=6n^2$ and $\left|E\left(G\right)\right|=3n\left(3n-1\right)$. In this structure there are three type of edges on the basis of their degrees, so we can decompose the the set of edges as $E\left(G\right)=E_1\left(G\right)\bigcup E_2\left(G\right)\bigcup E_3\left(G\right)$, where

$$\begin{aligned} E_1\left(G\right) &= \left\{e=rt\in E\left(G\right) \mid \xi_r=2,\xi_t=2\right\}, \\ E_2\left(G\right) &= \left\{e=rt\in E\left(G\right) \mid \xi_r=2,\xi_t=3\right\}, \\ E_3\left(G\right) &= \left\{e=rt\in E\left(G\right) \mid \xi_r=3,\xi_t=3\right\}. \end{aligned}$$

It is easy to check that $\left|E_1\left(G\right)\right|=6$, $\left|E_2\left(G\right)\right|=12\left(n-1\right)$ and $\left|E_3\left(G\right)\right|=3\left(3n^2-5n+2\right)$.

Theorem 32. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Honeycomb Network HC_n. Then the first multiplicative Zagreb index for G is $2^{3\left(3n^2-5n+6\right)}\times 3^{3\left(3n^2-5n+2\right)}\times 5^{12(n-1)}$.*

Theorem 33. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Honeycomb Network HC_n. Then the second multiplicative Zagreb index for G is $2^{12n}\times 3^{18n(n-1)}$.*

Theorem 34. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Honeycomb Network HC_n. Then the multiplicative atomic bond connectivity index for G is $2^{3\left(3n^2-7n+3\right)}\times 3^{-3\left(3n^2-5n+2\right)}$.*

Theorem 35. *Let $G\left(V\left(G\right),E\left(G\right)\right)$ be the graph of Honeycomb Network HC_n. Then the multiplicative geometric arithmetic index for G is $2^{18(n-1)}\times 3^{6(n-1)}\times 5^{-12(n-1)}$.*

Remark 1. We can compute the following easily by using Propositions 2 and 3.

i $SCII\left(TUZC_6\left[p,q\right]\right)=2^{\frac{p(2-3q)}{2}}\times 3^{\frac{-p(2-3q)}{2}}\times 5^{-2p}$.
ii $PCII\left(TUZC_6\left[p,q\right]\right)=2^{-2p}\times 3^{-3pq}$.
iii $SCII\left(TUAC_6\left[p,q\right]\right)=\sqrt{2^{-p(1+3q)}}\times\sqrt{3^{-p(3q-1)}}\times 5^{-p}$.
iv $PCII\left(TUAC_6\left[p,q\right]\right)=2^{-2p}\times 3^{-3pq}$.
v $SCII\left(CNC_5\left[n\right]\right)=\sqrt[4]{2^{-5(3n^2+n+4)}}\times\sqrt[4]{3^{-5n(3n+1)}}\times 5^{-5n}$.
vi $PCII\left(CNC_5\left[n\right]\right)=2^{-5(n+1)}\times\sqrt{3^{-15n(n+1)}}$.
vii $SCII\left(SL_n\right)=2^{-9n(2n-1)}\times 3^{-3n(9n+1)}$.
viii $PCII\left(SL_n\right)=2^{-9n(3n-1)}\times 3^{-36n^2}$.
ix $SCII\left(CS_n\right)=\sqrt{2^{-3n}}\times 3^{-(5n-1)}$.
x $PCII\left(CS_n\right)=2^{-3(n-1)}\times 3^{-6n}$.
xi $SCII\left(HX_n\right)=2^{-9\left(n^2-2n-1\right)}\times\sqrt{3^{-3(3n^2-11n+14)}}\times 5^{-6(n-2)}\times 7^{-6}$.
xii $PCII\left(HX_n\right)=2^{-3\left(3n^2-n-9\right)}\times 3^{-3\left(3n^2-9n+10\right)}$.
xiii $SCII\left(OX_n\right)=2^{-3n(9n-4)}\times 3^{-6n}$.
xiv $PCII\left(OX_n\right)=2^{-6n(6n-1)}$.
xv $SCII\left(HC_n\right)=\sqrt{2^{-3(3n^2-5n+6)}}\times\sqrt{3^{-3(3n^2-5n+2)}}\times 5^{-6(n-1)}$.
xvi $PCII\left(HC_n\right)=2^{-6n}\times 3^{-9n(n-1)}$.

Remark 2. We can compute the following easily by using Fact 1.

i $HII_1\left(TUZC_6\left[p,q\right]\right) = 2^{2p(3q-2)} \times 3^{2p(3q-2)} \times 5^{8p}$.
ii $HII_2\left(TUZC_6\left[p,q\right]\right) = 2^{8p} \times 3^{12pq}$.
iii $MZ_1^{\alpha}\left(TUZC_6\left[p,q\right]\right) = 2^{\alpha p(3q-2)} \times 3^{\alpha p(3q-2)} \times 5^{4\alpha p}$.
iv $MZ_2^{\alpha}\left(TUZC_6\left[p,q\right]\right) = 2^{4\alpha p} \times 3^{6\alpha pq}$.
v $G^*A^{\hat{\alpha}}II\left(TUZC_6\left[p,q\right]\right) = 2^{6\alpha p} \times 3^{2\alpha p} \times 5^{-4\alpha p}$.
vi $HII_1\left(TUAC_6\left[p,q\right]\right) = 2^{2p(1+3q)} \times 3^{2p(3q-1)} \times 5^{4p}$.
vii $HII_2\left(TUAC_6\left[p,q\right]\right) = 2^{8p} \times 3^{12pq}$.
viii $MZ_1^{\alpha}\left(TUAC_6\left[p,q\right]\right) = 2^{\alpha p(1+3q)} \times 3^{\alpha p(3q-1)} \times 5^{2\alpha p}$.
ix $MZ_2^{\alpha}\left(TUAC_6\left[p,q\right]\right) = 2^{4\alpha p} \times 3^{6\alpha pq}$.
x $G^*A^{\hat{\alpha}}II\left(TUAC_6\left[p,q\right]\right) = 2^{3\alpha p} \times 3^{\alpha p} \times 5^{-2\alpha p}$.
xi $HII_1\left(CNC_5\left[n\right]\right) = 2^{15n^2+5n+20} \times 3^{15n^2+5n} \times 5^{20n}$.
xii $HII_2\left(CNC_5\left[n\right]\right) = 2^{20(n+1)} \times 3^{30n(n+1)}$.
xiii $MZ_1^{\alpha}\left(CNC_5\left[n\right]\right) = \sqrt{2^{\alpha(15n^2+5n+20)}} \times \sqrt{3^{\alpha(15n^2+5n)}} \times 5^{10\alpha n}$.
xiv $MZ_2^{\alpha}\left(CNC_5\left[n\right]\right) = 2^{10\alpha(n+1)} \times 3^{15\alpha n(n+1)}$.
xv $G^*A^{\hat{\alpha}}II\left(CNC_5\left[n\right]\right) = 2^{15\alpha n} \times 3^{5\alpha n} \times 5^{-10\alpha n}$.
xvi $HII_1\left(SL_n\right) = 2^{36n(2n-1)} \times 3^{12n(9n+1)}$.
xvii $HII_2\left(SL_n\right) = 2^{36n(3n-1)} \times 3^{144n^2}$.
xviii $MZ_1^{\alpha}\left(SL_n\right) = 2^{18\alpha n(2n-1)} \times 3^{6n\alpha(9n+1)}$.
xix $MZ_2^{\alpha}\left(SL_n\right) = 2^{18\alpha n(3n-1)} \times 3^{72\alpha n^2}$.
xx $G^*A^{\hat{\alpha}}II\left(SL_n\right) = 2^{9\alpha n(3n+1)} \times 3^{-6\alpha n(3n+1)}$.
xxi $HII_1\left(CS_n\right) = 2^{6n} \times 3^{4(5n-1)}$.
xxii $HII_2\left(CS_n\right) = 2^{12(n-1)} \times 3^{24n}$.
xxiii $MZ_1^{\alpha}\left(CS_n\right) = 2^{3\alpha n} \times 3^{2\alpha(5n-1)}$.
xxiv $MZ_2^{\alpha}\left(CS_n\right) = 2^{6\alpha(n-1)} \times 3^{12\alpha n}$.
xxv $G^*A^{\hat{\alpha}}II\left(CS_n\right) = 2^{3\alpha(2n-1)} \times 3^{-2\alpha(2n-1)}$.
xxvi $HII_1\left(HX_n\right) = 2^{36\left(n^2-2n-1\right)} \times 3^{6\left(3n^2-11n+14\right)} \times 5^{24(n-2)} \times 7^{24}$.
xxvii $HII_2\left(HX_n\right) = 2^{12\left(3n^2-n-9\right)} \times 3^{12\left(3n^2-9n+10\right)}$.
xxviii $MZ_1^{\alpha}\left(HX_n\right) = 2^{18\alpha\left(n^2-2n-1\right)} \times 3^{3\alpha\left(3n^2-11n+14\right)} \times 5^{12\alpha(n-2)} \times 7^{12\alpha}$.
xxix $MZ_2^{\alpha}\left(HX_n\right) = 2^{6\alpha\left(3n^2-n-9\right)} \times 3^{6\alpha\left(3n^2-9n+10\right)}$.
xxx $G^*A^{\hat{\alpha}}II\left(HX_n\right) = 2^{3\alpha(6n-1)} \times 3^{6\alpha(n-2)} \times 5^{-12\alpha(n-2)} \times 7^{-12\alpha}$.
xxxi $HII_1\left(OX_n\right) = 2^{12n(9n-4)} \times 3^{24n}$.
xxxii $HII_2\left(OX_n\right) = 2^{24n(6n-1)}$.
xxxiii $MZ_1^{\alpha}\left(OX_n\right) = 2^{6\alpha n(9n-4)} \times 3^{12\alpha n}$.
xxxiv $MZ_2^{\alpha}\left(OX_n\right) = 2^{12\alpha n(6n-1)}$.
xxxv $G^*A^{\hat{\alpha}}II\left(OX_n\right) = 2^{18\alpha n} \times 3^{-12\alpha n}$.
xxxvi $HII_1\left(HC_n\right) = 2^{6\left(3n^2-5n+6\right)} \times 3^{6\left(3n^2-5n+2\right)} \times 5^{24(n-1)}$.
xxxvii $HII_2\left(HC_n\right) = 2^{24n} \times 3^{36n(n-1)}$.
xxxviii $MZ_1^{\alpha}\left(HC_n\right) = 2^{3\alpha\left(3n^2-5n+6\right)} \times 3^{3\alpha\left(3n^2-5n+2\right)} \times 5^{12\alpha(n-1)}$.
xxxix $MZ_2^{\alpha}\left(HC_n\right) = 2^{12\alpha n} \times 3^{18\alpha n(n-1)}$.
xxxx $G^*A^{\hat{\alpha}}II\left(HC_n\right) = 2^{18\alpha(n-1)} \times 3^{6\alpha(n-1)} \times 5^{-12\alpha(n-1)}$.

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